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Leptons and Leptonic Currents.

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In this letter I call leptons: the positive muon, the negative electron, and the neutrino. The neutrino is described by a four-component spinor. Leptons are conserved. Such a scheme is consistent with the results of the high-energy neutrino experiment carried out at Brookhaven (1), with the absence of $\mu \to e+\gamma$ (2), $\mu \to 3e$ (3), $\mu^- + \text{nucleus} \to e^- + \text{nucleus}$ (4), and forbids $e^- + \mu^+ \to \mu^- + e^+$ (5) (muonium-antimuonium conversion) and $e^- + e^- \to \mu^- + \mu^-$ (6).

We apply the group $U_3 = U_1 \times SU_2$ of unitary transformations (7) on the three leptons and classify particles and currents according to their behaviour under U_3 and one of its subgroup SU_2 (isotopic spin subgroup). The results are:

(i) *Currents.* The possible sets of currents are given in Table I*a* and I*b*. Such sets divide into two groups. Sets of the first group are composed of currents with a definite behaviour under parity. They can not give rise to parity nonconserving muon decay. Sets of the second group have the chiral nature of the $V-A$ theory For sets of the second group the charged current

$$(1)\qquad \begin{cases} \frac{1}{2}(j_1 - ij_2) = -\dfrac{i}{2}(\pm\bar{e}\gamma a v + \bar{\nu}\gamma\bar{a}\mu)\,, \\[2ex] \qquad\qquad\quad = -\dfrac{i}{2}(\pm\bar{e}\gamma v_e - \bar{\mu}^c\gamma v_\mu)\,, \end{cases}$$

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(3) S. Parker and S. Penman: *Nuovo Cimento*, **23**, 485 (1962); A. I. Alikhanov, A. T. Babaev, M. Ya Balats, V. S. Kaftanov, L. G. Landsberg, V. A. Lyubimov and Yu. V. Obukhov: *Proc. Intern. Conf. on High Energy Physics at CERN*, edited by J. Prentki (Geneva, 1962) p. 423.

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(6) N. Cabibbo and R. Gatto: *Nuovo Cimento*, **19**, 612 (1961).

(7) See: R. H. Behrends, J. Dreitlein, C. Fronsdal and W. Lee: *Rev. Mod. Phys.*, **34**, 1 (1962).

TABLE I*a*. – *Sets of the first group: positive helicity leptons and negative helicity leptons transform according to equivalent representations of* SU_3.

$\frac{1}{2}(j_1 + ij_2)$	$-(i/2)\bar{\mu}\gamma\nu$	$-(i/2)\bar{\mu}\gamma\nu$	$(i/2)\bar{\mu}\gamma\gamma_5\nu$	$(i/2)\bar{\mu}\gamma\gamma_5\nu$
j_3	$-(i/2)(\bar{\mu}\gamma\mu - \bar{\nu}\gamma\nu)$			
$\frac{1}{2}(j_4 + ij_5)$	$-(i/2)\bar{\mu}\gamma e$	$(i/2)\bar{\mu}\gamma\gamma_5 e$	$(i/2)\bar{\mu}\gamma\gamma_5 e$	$-(i/2)\bar{\mu}\gamma e$
$\frac{1}{2}(j_6 + ij_7)$	$-(i/2)\bar{\nu}\gamma e$	$(i/2)\bar{\nu}\gamma\gamma_5 e$	$-(i/2)\bar{\nu}\gamma e$	$(i/2)\bar{\nu}\gamma\gamma_5 e$
j_8	$-(i/2)(1/\sqrt{3})(\bar{\mu}\gamma\mu + \bar{\nu}\gamma\nu - 2\bar{e}\gamma e)$			

TABLE I*b*. – *Sets of the second group: positive helicity leptons and negative helicity leptons transform according to inequivalent representations of* SU_3.

$\frac{1}{2}(j_1+ij_2)$	$-(i/2)(\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-(i/2)(\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-(i/2)(-\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-(i/2)(-\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$
j_3	$-(i/2)(\bar{\nu}\gamma a\nu - \bar{e}\gamma ae+\bar{\mu}\gamma\bar{a}\mu - \bar{\nu}\gamma\bar{a}\nu)$			
$\frac{1}{2}(j_4+ij_5)$	$-(i/2)(-\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-(i/2)(\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-(i/2)(\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-(i/2)(-\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$
$\frac{1}{2}(j_6+ij_7)$	$-(i/2)(\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-(i/2)(-\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-(i/2)(\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-(i/2)(-\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$
j_8	$-(i/2)(1/\sqrt{3})(2\bar{\mu}\gamma a\mu - \bar{\nu}\gamma a\nu - \bar{e}\gamma ae+\bar{\mu}\gamma\bar{a}\mu+\bar{\nu}\gamma\bar{a}\nu - 2\bar{e}\gamma\bar{a}e)$			

is the experimentally established charged lepton current (and similarly for its charge-conjugate current $\frac{1}{2}(j_1 + ij_2)$). In (1) we have put

(2) $$a = \tfrac{1}{2}(1 + \gamma_5)\,, \qquad \bar{a} = \tfrac{1}{2}(1 - \gamma_5)$$

(2′) $$\nu_e = a\nu\,, \qquad \nu_\mu = a\nu^c\,.$$

We conclude that only sets of the second group can be physically significant, and we are led to a classification according to that subgroup SU_2 of SU_3, whose generators are obtained from $\frac{1}{2}(j_1 \pm ij_2)$ and j_3 (lepton isospin subgroup).

(ii) *Particles*. The choice, from experiment, of sets of the second group implies that the positive-helicity leptons (that we call: μ_+, ν_+, and e_+) and the negative-helicity leptons (that we call: μ_-, ν_-, and e_-) transform, under SU_3, according to inequivalent three-dimensional representations. This is illustrated in Fig. 1 where the weight-diagrams of the representations $D^3(1, 0)$ and $D^3(0, 1)$ are reported. We have chosen the positive-helicity leptons to transform according to $D^3(1, 0)$.

(iii) *Quantum numbers*. In the weight diagrams of Fig. 1, $(F_3^{(+)}, F_8^{(+)})$ and $(F_3^{(-)}, F_8^{(-)})$ are commuting group generators. We introduce

(3) $$I_3^{(+)} = F_3^{(+)}$$

(3′) $$S^{(+)} = 2\sqrt{3}F_8^{(+)} - L$$

(where L is the lepton number) and, similarly, $I_3^{(-)}$ and $S^{(-)}$. We can write for the charge Q:

(4) $$Q = I_3^{(+)} + \frac{L + S^{(+)}}{2} =$$

(4′) $$= I_3^{(-)} + \frac{L + S^{(-)}}{2},$$

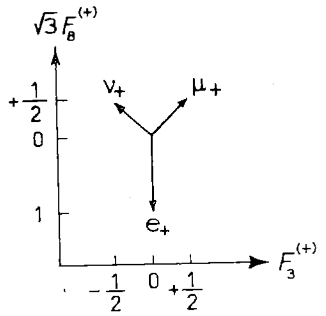

Fig. 1a. – Weight diagram for the positive helicity leptons.

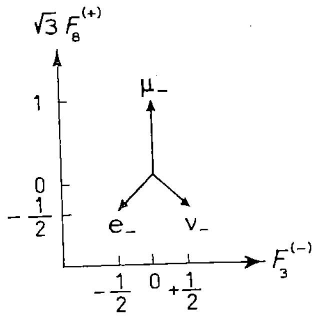

Fig. 1b. – Weight diagram for the negative helicity leptons.

and we can label particles and currents by their right-isospin $\boldsymbol{I}^{(+)}$, left-isospin $\boldsymbol{I}^{(-)}$, right-strangeness $S^{(+)}$ and left-strangeness $S^{(-)}$ (see Tables II*a*, II*b* and III).

TABLE II*a*. – *Quantum number assignments to the leptons with positive helicity.* The corresponding baryons are indicated in last column. The baryon Z^- has not yet been discovered.

Particle	Lepton number L	Charge Q	$S^{(+)}$	$\lvert \boldsymbol{I}^{(+)} \rvert$	$I_3^{(+)}$	Corresponding baryon
μ_+	$+1$	$+1$	0	$\frac{1}{2}$	$\frac{1}{2}$	p
ν_+	$+1$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	n
e_+	$+1$	-1	-3	0	0	Z^-

TABLE II*b*. – *Quantum number assignments to the leptons with negative helicity* The corresponding baryons are indicated in the last column. The baryon X^+ has not yet been discovered.

Particle	Lepton number L	Charge Q	$S^{(-)}$	$\lvert \boldsymbol{I}^{(-)} \rvert$	$I_3^{(-)}$	Corresponding baryon
μ_-	$+1$	$+1$	1	0	0	X^+
ν_-	$+1$	0	-2	$\frac{1}{2}$	$\frac{1}{2}$	Ξ^0
e_-	$+1$	-1	-2	$\frac{1}{2}$	$-\frac{1}{2}$	Ξ^-

TABLE III. – *Quantum number assignments to the current.* The corresponding (vector) mesons are indicated in the last column. The mesons φ have not yet been discovered.

Current	Lepton number L	Charge Q	$S^{(-)}=S^{(+)}$	$\lvert \boldsymbol{I}^{(-)} \rvert = \lvert \boldsymbol{I}^{(+)} \rvert$	$I_3^{(-)}=I_3^{(+)}$	Corresponding meson
$\frac{1}{2}(j_1 \pm ij_2)$	0	± 1	0	1	± 1	ρ^+, ρ^-
j_3	0	0	0	1	0	ρ_0
$\frac{1}{2}(j_4 \pm ij_5)$	0	± 2	± 3	$\frac{1}{2}$	$\pm\frac{1}{2}$	φ^{++}, φ^{--}
$\frac{1}{2}(j_6 \pm ij_7)$	0	± 1	± 3	$\frac{1}{2}$	$\pm\frac{1}{2}$	φ^+, φ^-
j_8	0	0	0	0	0	ω_0

(iv) *Baryon-lepton symmetry.* In the last columns of the Tables II*a*, II*b* and III we have reported the « corresponding baryon » and the « corresponding meson » for each particle and current, *i.e.* the baryon or boson with corresponding quantum numbers (the correspondence is: lepton number ↔ nucleon number; $Q \leftrightarrow Q$; $S^{(+)}$, $S^{(-)} \leftrightarrow S$; $\boldsymbol{I}^{(+)}$, $\boldsymbol{I}^{(-)} \leftrightarrow \boldsymbol{I}$). The baryons Z^- (isotopic spin $I=0$, and strangeness $S=-3$) and X^+ ($I=0$, $S=+1$), and the mesons φ ($I=\frac{1}{2}$, $S=\pm 3$) have not been found so far. The baryon-lepton correspondence rules of Tables II*a*, II*b* and III replace the GAMBA, OKUBO, MARSHAK correspondence rule $p \leftrightarrow \nu$, $n \leftrightarrow e^-$, $\Lambda \leftrightarrow \mu^-$ (8).

(v) *Couplings.* Invariance of the weak four-lepton coupling under the full unitary group can be excluded: it would lead to a parity-conserving interaction. We generate the weak four-lepton Lagrangian, L', by self-coupling of the current of Table I*b* (for each of the chosen sets of the second group). We write

$$L' = gL_1 + fL_2 + hL_3 \tag{5}$$

where L_1, L_2, and L_3 are invariant under the lepton-isospin subgroup SU_2. L_1 arises from the self-coupling of j_1, j_2, j_3; L_2 from the self-coupling of j_4, j_5, j_6, j_7; and L_3 from the self-coupling of j_8. In the limit of unitary symmetry $g=f=h$. From the measured values of the ξ parameter in μ-decay we can show that

$$f < 0.2g\,. \tag{6}$$

The result (6) indicates that the self-coupling of j_4, j_5, j_6, j_7 is presumably absent. Such coupling, if mediated by vector bosons, would have required double charged bosons. The coupling of j_6 and j_7 to strong interacting currents (both strangeness-conserving and strangeness-nonconserving) seems to be experimentally excluded (1,9).

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(9) G. FEINBERG and F. GÜRSEY and A. PAIS: *Phys. Rev. Lett.*, **7**, 208 (1961); S. BLUDMAN: *Phys. Rev.*, **124**, 947 (1961).

Invariance under SU_2 of the weak four-lepton interaction can directly be checked by measurement of the cross-section for scattering of ν_e on e (with neutrinos from nuclear reactors). It also allows to predict scattering of ν_μ on e and weak effects in scattering and μ-pair production in high-energy electron-positron colliding beams.

We shall now sketch briefly the main lines of the argument. The generators of the unitary group are called $F_0 = L, F_1, F_2, \ldots, F_8$, and are assumed to be integrals of partially conserved local currents

$$F_i = -i\int j_i^\mu(x)\,\mathrm{d}\sigma_\mu\,. \tag{7}$$

The current j_i, because of its vector character, can be decomposed into a contribution from positive-helicity particles and one from negative-helicity particles. Correspondingly

$$F_i = F_i^{(+)} + F_i^{(-)} \tag{8}$$

and $F_i^{(+)}$, $F_i^{(-)}$ satisfy the commutation relations

$$[F_i^{(+)}, F_j^{(+)}] = if_{ijk}F_i^{(+)}\,, \tag{9}$$

$$[F_i^{(-)}, F_j^{(-)}] = if_{ijk}F_i^{(-)}\,. \tag{9'}$$

The commutation relations with the charge operator Q must be of the form

$$[Q, F_i^{(+)}] = c_{ik}F_k^{(+)}\,, \tag{10}$$

$$[Q, F_i^{(-)}] = c_{ik}F_k^{(-)}\,. \tag{11}$$

We do not need to specify c_{ik}.

We now construct the 3-dimensional representations f_i of F_i. We distinguish two cases:

1) $f_i^{(-)}$ and $f_i^{(+)}$ are related by a similarity transformation

$$f_i^{(-)} = wf_i^{(+)}w^{-1}; \tag{12}$$

2) $f_i^{(-)}$ and $f_i^{(+)}$ are not related by a similarity transformation. In such a case

$$f_i^{(-)} = w\tilde{f}_i^{(+)}w^{-1} \tag{13}$$

must hold, where $\tilde{f}_i^{(+)}$ is the representation contragredient to $f_i^{(+)}$. Using (10) and (11) we can show that: in case 1)

$$[q, w] = 0 \tag{14}$$

while in case 2)

$$\{q, w\} = 0 \tag{15}$$

where $\{\ldots\}$ denotes the anticommutator and q is the 3×3 representation of Q. In both cases, (14) or (15) define w in terms of two real parameters. Having constructed $f_i^{(-)}$ we demand for it the same reality properties of $f_i^{(+)}$ (time reversal invariance). Such a procedure leads directly to the currents of Table I.

The Lagrangians L_1, L_3 are given by

$$L_1 = \pm(\bar{\nu}\gamma ae)(\bar{\nu}\gamma\bar{a}\mu) \pm (\bar{\mu}\gamma\bar{a}\nu)(\bar{e}\gamma a\nu) + \tfrac{1}{4}(\bar{\nu}\gamma\nu)(\bar{e}\gamma ae) + \tfrac{1}{4}(\bar{\nu}\gamma\nu)(\bar{\mu}\gamma\bar{a}\mu) - \\ - \tfrac{1}{2}(\bar{e}\gamma ae)(\bar{\mu}\gamma\bar{a}\mu) + \tfrac{1}{4}(\bar{e}\gamma ae)(\bar{e}\gamma ae) + \tfrac{1}{4}(\bar{\mu}\gamma a\mu)(\bar{\mu}\gamma\bar{a}\mu) + \tfrac{1}{4}(\bar{\nu}\gamma\gamma_5\nu)(\bar{\nu}\gamma\gamma_5\nu)\,,$$

$$L_3 = \tfrac{1}{12}\,[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)][2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)] + \\ + \tfrac{1}{12}\,[2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)][2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)] + \\ + \tfrac{1}{12}\,(\bar{\nu}\gamma\gamma_5\nu)(\bar{\nu}\gamma\gamma_5\nu) + \tfrac{1}{6}\,[2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)](\bar{\nu}\gamma\gamma_5\nu) - \\ - \tfrac{1}{6}\,[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)](\bar{\nu}\gamma\gamma_5\nu) - \tfrac{1}{6}\,[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)][2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)]\,.$$

We do not report the form of L_2, since as we have indicated it is presumably absent. Both L_1 and L_2 would contribute to μ-decay. The total contribution would be of the form

$$(16)\qquad \big(\bar{e}\gamma(q + p\gamma_5)\mu^c\big)(\bar{\nu}^c\gamma\gamma_5\nu)\,.$$

If L_2 is absent, $p = q$ and (16) becomes the known μ-decay Lagrangian, written in a simple form that nobody uses. The physical consequences of (16) can be simply read off from on old-fashioned paper on Pauli-Pursey invariants in μ-decay (10). It gives for the muons decay parameters $\varrho = \frac{3}{4}$, $\delta = \frac{3}{4}$, and $\xi = -2pq/(p^2 + q^2)$. Using Steinberger's figure (11) for ξ we have obtained (5).

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(10) R. GATTO and G. LÜDERS: *Nuovo Cimento*, **7**, 806 (1958).
(11) J. STEINBERGER: *Rendiconti S.I.F.*, Corso XI (Varenna, 1959), p. 375.